MATH 105 101 Midterm 1 Sample 3

1. (15 marks)

   (a) (3 marks) Let

   $$f(x,y) = y^2 + y \ln x.$$

   Compute both first-order partial derivatives of f at the point $(1,2)$. *Simplify your answers.*

   (b) (2 marks) Given $f(x,y)$ as in part (a), sketch the trace of the surface $z = f(x,y)$ in the $x = 1$ plane.

   (c) (2 marks) Find a unit vector parallel to $\langle -2, 1, 2\rangle$.

   (d) (2 marks) Find an equation for the plane passing through the point $P(1,2,3)$ that is orthogonal to the vector $\langle 4, 0, -1\rangle$.

   (e) (3 marks) Determine if the plane described by the equation:

   $$2x - 5y + 2z = -1,$$

   is orthogonal to the plane given in part (d).

   (f) (3 marks) Assume that $f(x,y)$ has continuous partial derivatives of all orders. If

   $$f_y(x,y) = x^3 + 2x^2 y,$$

   compute f_{xyx}. State in detail any result that you use.

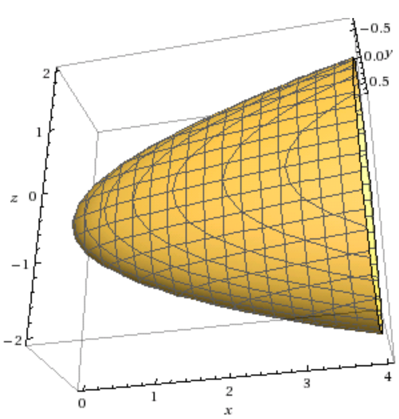
2. (5 marks) Consider the surface S given by:

   $$z^2 = x - 9y^2.$$

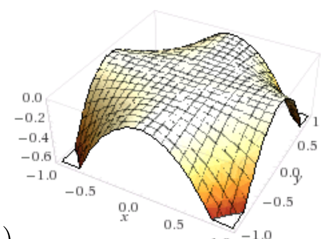
   (a) (4 marks) Find and sketch the level curves of S for $z_0 = 1$ and $z_0 = 2$.

   (b) (1 mark) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?

(A)



(B)



3. (10 marks) Let R be the semicircular region $\{x^2 + y^2 \leq 4, x \geq 0\}$. Find the maximum and minimum values of the function

$$f(x, y) = x^2 - 2x + y^2.$$

on the *boundary of the region R.*

4. (10 marks) Find *all* critical points of the following function:

$$f(x, y) = xy - \frac{x^2}{2} - \frac{y^3}{3} + 5.$$

Classify each point as a local minimum, local maximum, or saddle point.

5. (10 marks) A firm produces:

$$P(x, y) = x^{\frac{2}{3}} y^{\frac{1}{3}}$$

units of goods per week, utilizing x units of labour and y units of capital. If labour costs \$27 per unit, and capital costs \$0.5 per unit, use the method of Lagrange multiplier to find the most cost-efficient division of labour and capital that the firm can adopt if its goal is to produce 6 units of goods per week.

Clearly state the objective function and the constraint. *You are not required to justify that the solution you obtained is the absolute maximum.* **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**